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FERTILITY PROJECTIONS: PEARSON TYPE III CURVE VS 3-YEAR AVERAGE AGE-SPECIFIC FERTILITY RATES*

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ABSTRACT

The purpose of this paper is to compare the Pearson Type III curve method with the constant age-specific fertility rate method for fertility projection. The projected number of births derived by the two methods are compared at the provincial level for the years 1993 to 2001. An analysis of the results shows that the differences in the projected number of births generated by these methods are not large. However, the Pearson Type III Curve is a more analytically powerful method than the constant age-specific fertility rate method for projecting births.

KEY WORDS: Parametric model; Pearson Type III curve; Constant age-specific fertility rate method.

RÉSUMÉ

Afin d'évaluer les mérites relatifs de la méthode de la courbe de Type III de Pearson et de celle du taux constant de fécondité par âge pour établir des projections concernant la fécondité, cette étude compare les nombres projetés de naissances obtenus par chaque méthode, au niveau provincial, pour la période de 1993 à 2001. L'analyse des résultats indique que les écarts entre les nombres projetés de naissances produits par les deux méthodes sont faibles, mais que, sur le plan analytique, la courbe de Type III de Pearson est une méthode plus puissante que celle du taux constant de fécondité par âge.

MOTS CLÉS: Modèle paramétrique; courbe de Type III de Pearson; méthode du taux constant de fécondité par âge.

1. INTRODUCTION

The projection of population by age and sex using the cohort-component method requires the projected number of births in a given year. For this, one has to multiply the projected number of women at each age by the fertility rate of the corresponding age, and then sum the products obtained. In recent years, Canadian data on total fertility rate, mean age of fertility, the variance and the third moment of the fertility distribution or skewness have been varying within a small range (Statistics Canada, 1994). Consequently, the future use of the Pearson Type III curve in projecting the number of births for Canada, provinces and territories was warranted. In 1993, Statistics Canada implemented the Pearson Type III over the Pearson Type I to graduate age-specific fertility rates. The purpose of this paper is to compare the projected number of births derived by the Pearson Type III curve with those generated by assuming a constant schedule of age-specific fertility rates. The comparisons are performed at the provincial levels and for the years 1993-1994 to 2000-2001.

2. LITERATURE REVIEW

Statistics Canada uses a parametric model to project the number of births in a given year for Canada, provinces and territories. In the past, the Pearson Type I curve was considered the most suitable method to project age-specific fertility rates as the mean age of fertility had consistently been higher than the modal age (Romaniuk, 1975). However, in recent years, the differences between the mean and modal ages of fertility have been narrowing and the shape of the distribution of childbearing is becoming more symmetrical. Based on analysis of birth data for the period 1971 to 1989, it was decided to use the Pearson

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Type III curve, instead of the Type I, to project the age-specific fertility rates for Canada, provinces and territories for the 1993-based population projections (Verma and Ford, 1992). This is because the Type III curve better portrays both the distribution of the age-specific fertility rates and the estimates of births.

3. PROJECTION METHODOLOGY AND ASSUMPTIONS

3.1 Pearson Type III Curve

The density function of the Pearson Type III curve (Elderton, 1930) is as follows:

$$f(x) = y_0\left(1 + \frac{x}{a}\right)^{\gamma a} e^{-\gamma x}$$

where x is measured as the deviation from the mode. The parameters, γ and a are calculated as follows:

$$\gamma = \frac{2\mu_2}{\mu_3},$$

$$a = \frac{2\mu_2^2}{\mu_3} - \frac{\mu_3}{2\mu_2}$$

and,

$$Mode = Mean - \frac{1}{\gamma}.$$

In order to apply the Type III model to project the age-specific fertility rate, projections of its four parameters, namely, the total fertility rate (TFR), mean age of fertility (μ_1), the variance (μ_2), and the third moment of the fertility distribution (μ_3), must be developed first. The first parameter provides a convenient measure of the level of fertility, while the latter three provide a measure of the age pattern of childbearing.

3.2 Constant Age-Specific Fertility Rates (ASFR) Method

In the constant ASFR method, it is assumed that the age schedule of fertility in the last available three years (1988 to 1990) will remain constant over the projection period. One could also assume a variable pattern of ASFRs during the projection years, but we have not considered this variation in the present study. The assumptions concerning future fertility are stated in terms of total fertility rates. The procedure (Shryock and Siegel, 1976) involves the following steps: (1) converting the schedules of 5-year age group age-specific fertility rates for the years 1988 to 1990 into percentage distributions; (2) taking an average of these three-year percentage distributions and assuming that this average distribution will remain constant over the projection period; (3) applying the projected TFR to the 3-year average percentage distribution to obtain projected age-specific fertility rates for the projected years; and (4) multiplying the projected age-specific fertility rates by the corresponding projected female population to obtain the projected number of births for the future years.

3.3 Assumptions on the Fertility Parameters

The three assumptions on total fertility rates and mean ages of fertility depicting low, medium, and high fertility variants are taken from the 1993-based population projections[2]. In these assumptions, it is assumed that mean age of fertility is inversely related to total fertility rate. At the national level, the assumptions are as follows:

Low assumption: The total fertility rate for Canada continues to decline from 1.70 births per woman in 1993 to 1.53 by 2001. This assumption is combined with a high variant for the mean age of fertility which is assumed to increase from 27.94 in 1993 to 28.11 by 2001.

Medium assumption: The total fertility rate is assumed to remain constant at 1.70 births per woman throughout the projection period. The mean age of fertility is assumed to change slightly from 27.94 in 1993 to 27.97 by 2001.

High assumption: The total fertility rate for Canada will increase from 1.70 in 1993 to 1.87 births per woman by 2001. This assumption is combined with a low variant for the mean age of fertility decreasing from 27.94 in 1993 to 27.83 by 2001.

The assumptions on total fertility rates and mean ages of fertility according to the three fertility variants for eight selected provinces are presented in Table 1.

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2 In the 1993-based population projections, the fertility assumptions were formulated up to the year 2016. The present study concentrates only on the projection period of 1993 to 2001.

Table 1: Total Fertility Rates and Mean Ages of Fertility, Selected Provinces, 1993 and 2001.

	N.S.	N.B.	QUE.	ONT.	MAN.	SASK.	ALTA.	B.C.
Total Fertility Rates								
1993	1.60	1.56	1.61	1.68	1.98	2.13	1.88	1.66
2001- Low	1.43	1.39	1.47	1.50	1.76	1.87	1.69	1.50
2001- Medium	1.59	1.55	1.64	1.67	1.96	2.08	1.88	1.67
2001- High	1.75	1.71	1.80	1.84	2.16	2.29	2.07	1.83
Mean Age of Fertility								
1993	27.34	26.74	27.88	28.50	27.27	26.80	27.51	28.11
2001- Low	27.50	26.90	28.05	28.67	27.43	26.96	27.68	28.28
2001- Medium	27.37	26.77	27.91	28.52	27.30	26.82	27.54	28.14
2001- High	27.23	26.63	27.77	28.38	27.16	26.69	27.40	28.00

Source: Statistics Canada, *Population Projections for Canada, Provinces and Territories, 1993-2016* (Catalogue No. 91-520 Occasional), 1994.

The projected total fertility rates and mean ages of fertility for the intervening years were obtained by interpolation (see Statistics Canada, 1994; Verma, Loh, Dai and Ford, 1994). In the case of the other two fertility parameters, variance and skewness, values are assumed constant over the projection period using a three-year average (1990, 1991 and 1992) of provincial or territorial levels.

3.4 Projected Female Population

The projected female population of reproductive ages used in this study was taken from the 1993-based population projections. The projected female population used in the low fertility assumption was taken from Scenario 1, the medium fertility assumption from Scenario 2, and the high fertility assumption from Scenario 3. A detailed description of the assumptions on mortality, immigration, emigration, internal migration, non-permanent residents and returning Canadians adopted in these three growth scenarios is given in Statistics Canada, 1994, Catalogue no. 91-520. The assumptions on mortality, immigration, and internal migration are different in the three growth scenarios. Hence, comparison of number of births between the two methods within the same scenario is done in this paper.

4. ANALYSIS OF RESULTS

A comparison of the projected number of births generated by the Pearson Type III curve and the constant ASFR method for the eight provinces and their total according to the three fertility assumptions is presented in Table 2. On the whole, the number of births projected by the parametric method is larger than that generated by the constant ASFR method throughout the projection period. For the total period, in comparison to the constant ASFR method, the parametric model produces 14,300 more births according to the low fertility assumption, and 13,700 and 12,000 more births according to the medium and high fertility assumptions, respectively.

The trend in the fertility difference between the two methods is not uniform across the provinces. For Ontario, the sum of the absolute difference over the eight-year projection period is the largest among the provinces: 10,300, 11,900 and 9,600 for the low, medium and high fertility assumptions, respectively. In contrast, the sum of the absolute difference is less than 1,000 for the provinces of Nova Scotia, New Brunswick, Manitoba and Alberta. The sum of the absolute difference between the two methods is around 1,000 for Saskatchewan for the three fertility assumptions. For Quebec, the Pearson Type III method produces 1,800, 1,500 and 2,300 more births according to the low, medium, and high fertility assumptions, respectively. The corresponding sums of the difference for British Columbia are 2,500, 1,700 and 1,700, respectively.

This pattern of provincial differences is likely to be related to the mean age of fertility. In 1993, among all the provinces, Ontario had the highest mean age of childbearing (28.50 years), which was more than half a year older than the national average age of childbearing of 27.94 years. Ontario was followed by British Columbia with a mean age of childbearing of 28.11 years. In the same year, New Brunswick had the youngest mean age of fertility of 26.74 years, with Saskatchewan (26.80) trailing behind.

The above comparisons are based on short term projections, up to the year 2001, in order to avoid the echo effects of the fertility assumptions. From 2008 onwards, babies born during the projection years will be entering the reproductive ages and having an echo effect on the projection results. Thus, the fertility differences will be larger for longer term projections.

5. CONCLUSION AND DISCUSSION

The magnitude of the differences in the projected number of births derived respectively by the two methods is small. It is debatable whether we should continue using the Pearson Type III curve which requires developing assumptions on four fertility parameters or whether we should use the constant ASFR method which requires only the assumption of one fertility parameter. The advantages of employing the Type III curve in fertility projections over the constant ASFR method are discussed in the following paragraphs.

The advantages of the parametric method lie in its analytical powers in developing the assumptions for the fertility parameters used in the model. In the present case, the four parameters are simple and appropriate for in-depth analysis in order to provide rationales for the assumptions. As mentioned by Ryder (1993), "the advantage of parameterizing comes from analytic insight, and from a sense of how the (age) distribution (of fertility) responds to change in the parameters." Such strength is not associated with the constant ASFR method.

The ASFR method has the advantage of being simple to implement and has the ability to incorporate assumptions on the TFR which has been shown to be the most important variable in estimating the future number of births. It is important to bear in mind that random fluctuations in the age pattern of fertility may be extended into the future if the constant ASFR method is used. In addition, its effectiveness as a projection model will be greatly affected if an anomaly occurs at the ages where the number of the females is also large.

In discussing fertility projection methods, Pittenger (1976, p.162) states that "static fertility projection schedules are not recommended other than for analytical, baseline projections". Recognizing that states and smaller areas have plenty of potential for variability through population "character" changes (caused, in part, by net change due to migration), a prudent model builder should design some flexibility into his (fertility) projection system.

Table 2: Cumulative Difference in the Projected Number of Births Generated by the Pearson Type III Curve and Constant ASFR Method, Selected Provinces, 1993-1994 to 2000-2001.

	Low Fertility Assumption			Medium Fertility Assumption			High Fertility Assumption		
	Type III	ASFR	DIFF	Type III	ASFR	DIFF	Type III	ASFR	DIFF
				(in thousands)					
Total	2781.8	2767.5	-14.3	2990.1	2976.4	-13.7	3194.4	3182.4	-12.0
N.S.	82.0	81.7	-0.3	87.6	87.5	-0.1	94.3	94.2	-0.1
N.B.	65.0	65.0	0.0	69.6	69.7	0.1	75.1	75.3	0.2
Que.	651.2	649.4	-1.8	699.0	697.5	-1.5	742.3	740.0	-2.3
Ont.	1103.8	1093.5	-10.3	1188.8	1176.9	-11.9	1267.2	1257.6	-9.6
Man.	118.8	118.9	0.1	127.4	127.6	0.2	133.9	134.2	0.3
Sask.	105.0	105.8	0.8	112.4	113.3	0.9	116.3	117.3	1.0
Alta.	301.9	301.6	-0.3	324.3	324.6	0.3	352.3	352.5	0.2
B.C.	354.1	351.6	-2.5	381.0	379.3	-1.7	413.0	411.3	-1.7

Note: Difference = Constant ASFR method - Pearson Type III method.
Source: Statistics Canada, Demography Division, Population Projections Section.

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