Birth-death processes (BDPs) are continuous-time Markov chains that are widely used as models for stochastic counting processes in molecular evolution. However, estimating mutation parameters in these models is remarkably difficult when observation of the process is discrete since likelihoods cannot be computed analytically. We derive EM algorithms for estimating mutation rates in BDPs and show that the E-step can be expressed as a convolution of transition probabilities for any general BDP with arbitrary rates. We use a continued fraction representation of the Laplace transforms of the transition probabilities to derive fast EM algorithms. This allows rapid estimation in a large class of previously intractable stochastic processes. We apply the method to estimate of mutation parameters in microsatellite evolution.