A Law of the Single Logarithm for Weighted Sums of Arrays Applied to Bootstrap Model Selection in Regression

We generalize a law of the single logarithm obtained by Qi (1994) and Li et al. (1995) to the case of weighted sums of triangular arrays of random variables. We apply this result to bootstrapping the all-subsets model selection problem in regression, where we show that the popular Bayesian Information Criterion of Schwarz (1978) is no longer asymptotically consistent. Indeed, at the bootstrap level, the weighted sum of random variables used to select the variables is of order \((n \log n)^{1/2}\) instead of \((n \log \log n)^{1/2}\) as with the original variables.